# Probabilistic Modelling and Analysis of a Fish Population ${ }^{\star}$ 

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#### Abstract

The fish stock of the common sole in the Adriatic Sea has been analysed by agent-based modelling and simulation techniques as an integration of other classical stock assessment models. In this work we start investigating also about the formal probabilistic modelling of our case study in order to extract valuable biological information from available formal verification techniques. In particular, a PRISM model for the common sole is developed and some initial results are discussed.


Keywords: Fish Stock Assessment, Common Sole, Adriatic Sea, Probabilistic Models, PRISM Model Checker.

## 1 Introduction

One of the most critical aspects about marine ecosystems and fish population nowadays is the sustainability of the fisheries. Fish stocks, even if they are renewable, are not unlimited. In a lot of cases, where the fishing activity is not controlled, fish stocks are overfished. In 2002 the World Summit on Sustainable Development (WSSD) gave guidelines on how to handle the marine ecosystem in the future [13]. In the European continent the management of fisheries is regulated by the Common Fisheries Policy (CFP) whose most recent version took effect on 1st January 2014 [14]. It stipulates that, between 2015 and 2020, catches should achieve a sustainable level for maintaining fish stocks in the long term. Until now, the policy did not have a big impact on the exploitation and its goal is still far to be achieved, especially in the Mediterranean Sea [15].

Some of the authors have been working on models for fish stock evaluation since 2010. The starting idea was to introduce agent-based models as an alternative for the ODE/PDE-based deterministic mathematical models usually

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Fig. 1. Workflow of the studies on the common sole stock: in green the agent-based modelling and simulation approach (past and present), in blue the model checking approach (present and future), in orange the statistical model checking approach (future).
employed in the field [2]. The use of agent-based models or, more generally, of individual-based, compositional and stochastic models was, and still is, a promising approach in the context of ecological modelling [4]. The fish stock on which we have concentrated our efforts is the one of the common sole, Solea solea (Linnaeus, 1758), in the northern and central Adriatic Sea. Independent scientific surveys in this area have been performed since 2005 by the Solemon Project [ 3 , 10], which allows us to have a suitable collection of data for validation.

In [1] an automata-based formalism was introduced, namely Extended Probabilistic Discrete Timed Automata (EPDTA). This formalism permitted to easily model the time-dependent and probabilistic basic behaviour of a common sole and to translate it directly into a Markov decision process. This EPDTA model was used as a basis for defining the behaviour of an individual agent, representing a sole, in the development and validation of the version 1.0 of the Demersal fIsh Stock Probabilistic Agent-based Simulator (DISPAS) [9, 8]. The version 2.0 of the simulator, which scales from an average square kilometre of Adriatic Sea to the whole surface of the sea, was presented in [7] and is under development.

Figure 1 shows the workflow of the studies that we performed so far, which can be considered an example framework for non-marine biologists who are interested in a combination of methods to "simulate and test". Starting from data, the EPDTA model is derived. On the left (green) side we used it to derive the probabilistic agent-based model of DISPAS, which required additional data for handling newborns. The simulated data were statistically analysed and compared with real data for validation $[9,8]$. The validated model can be used to get new information by simulating different future scenarios with different levels of fishing efforts or different environmental conditions.


Fig. 2. Part of an EPDTA representing the behaviour of a sole in class $i$. The double circled state is the initial one when $i=0$. From state chkL $i$ the automaton goes to the next class $i+1$ if the length of the sole is sufficient to be considered in the new class.

In this work we present the initial results of the right (blue) part of the flow. Starting from the same EPDTA model, a more analytical approach is followed. The model is translated into a Discrete Time Markov Chain (DTMC), or another stochastic formalism, and is given as input to a model checker in order to be verified against formally expressed properties. The results of this kind of analysis can be validated with the available data and can also be compared with those obtained using the simulation-based approach. In particular, in this work we report on the creation of a DTMC, derived from our initial EPDTA model, that can be given as input to the PRISM model checker and we report some initial results obtained verifying properties expressed in Probabilistic Computation Tree Logic (PCTL) [5]. The far right (orange) part of the flow shows a planned future work in which the stochastic model is used to verify formal properties by statistical model checking [6] using tools, among which PRISM itself, that are now sufficient mature in this relatively new area of semi-formal analysis.

## 2 Sole Behaviour as DTMC

Figure 2 shows a part of an EPDTA representing the probabilistic and timed behaviour of a sole $[9,8]$. Soles are divided in classes according to their length, assuming that individuals in the same class are subject to the same probability of being fished or of dying for natural mortality. The clock $x$ is discrete-time and measures time in months. The length of the sole is updated each month according to its age and to environmental conditions summarised by a constant $k$ in the von Bertanlaffy growth function [12]. Then, in state chкM $i$, the sole can die for natural mortality (predators, availability of food, environmental conditions) with a certain probability $\operatorname{Pr} M(i, t)$ depending on the class $i$ and on the current month $t$. If the sole survives, in state chkF $i$ the sole can be fished with a certain probability $\operatorname{Pr} F(i, t)$. If the sole survives then it proceeds to the next month possibly changing class if a given threshold on the length is reached.

The crucial point of this relatively simple model is the estimation of the probabilities for each class. In order to do so we used the natural mortality index $M$ and the fishing effort index $F$ calculated in the framework of specific working groups for the assessment of demersal stocks [10]. These indices represent the annual exponential decay, $e^{-(M+F)}$, of the population of a class according to the two causes of mortality. We took the indexes for each class from year 2006 to year 2013 and calculated the annual fishing mortality probability of each year $y$ as $\operatorname{Pr} F(i, y)=1-e^{-F_{y}}$. To distribute the probability among each month, according to the Markovian characteristic of the model, we simply divided the annual probability by 12 , i.e., $\operatorname{PrF}(i, t)=\operatorname{PrF}(i, y) / 12$. It must be mentioned that, according to information on fishery patterns or on fishing bans, this distribution over months can be refined considerably. The probability that quantifies natural mortality per year was calculated in the same way but, for the sake of simplicity, it was considered constant over the years. The following table shows the values used in the model. Column PrM reports the annual constant mortality probabilities while the other columns report the values of annual PrF .

| Cl | PrM | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0.50341 | 0.18149 | 0.15970 | 0.14053 | 0.20768 | 0.15274 | 0.13822 | 0.15597 | 0.07672 |
| 1 | 0.29531 | 0.62506 | 0.56932 | 0.52251 | 0.67468 | 0.55412 | 0.52237 | 0.57209 | 0.32942 |
| 2 | 0.24421 | 0.36292 | 0.32834 | 0.27957 | 0.41247 | 0.31358 | 0.30154 | 0.30531 | 0.15760 |
| 3 | 0.22119 | 0.33458 | 0.30895 | 0.23496 | 0.38797 | 0.29187 | 0.31524 | 0.25808 | 0.13110 |
| 4 | 0.20546 | 0.28084 | 0.25701 | 0.20232 | 0.32702 | 0.24283 | 0.25029 | 0.22054 | 0.11068 |
| $5+$ | 0.19748 | 0.24615 | 0.22363 | 0.18184 | 0.28713 | 0.21144 | 0.20794 | 0.19685 | 0.09805 |

In order to translate the EPDTA model into the PRISM model we proceeded ideally as formally specified in the semantics of EPDTA [1], with some optimisations. In particular, we used PRISM integer variables month and year to simulate the discrete clock $x$ in the EPDTA model and we simulated the passage of time by inserting periodical transitions suitably incrementing these variables. Moreover, in the particular EPDTA model of the sole behaviour, non-determinism does not arise, thus yielding a DTMC instead of a Markov decision process, which is, in general, the formal semantics of an EPDTA. All the probabilities in the PRISM model were expressed as constants and additional states were added to allow the time elapse after 2013 or after the death of the sole ${ }^{5}$.

## 3 Discussion and Conclusions

The first formal analysis done on the model was to determine the survival trend of the sole during the whole period (from 2006 to 2013). This was done by exploiting the PRISM capability of calculating the probability of a given PCTL formula. In particular, the request

$$
\mathrm{P}=?[\mathrm{~F}!\text { state }=4 \&!\text { state }=3 \& \text { year }=y \& \text { month }=m]
$$

[^1]makes PRISM calculate the probability that a state in which the sole is alive, i.e. not fished (state 3) and not naturally dead (state 4), is reachable when the current year is $y$ and the current month is $m$. Figure 3 shows the values calculated making the variables $y$ and $m$ vary on the period 2006-2013.


Fig. 3. Plot of the survival probabilities calculated from the PRISM model. On the $x$ axis the months over the period 2006-2013.

A first interesting evaluation of this result is that the probability of survival becomes less than 0.5 just after 12 months and becomes less than 0.25 after 22 months. This is in accordance to the evaluation of fishery biologists that the stock under consideration is not only overfished, but also there is a huge pressure on juveniles which compromises the stock capacity of replenishment.

Another comparison of this result with the information coming from the same data but from other models is about longevity. Longevity is the maximum age reported in years that individuals of a given population would reach. Following [11], it can be calculated as the age at $95 \%$ of the asymptotic length of the fish, a parameter of the von Bertalanffy growth equation. Using Solemon data and this evaluation model, the longevity of the sole in the considered stock was estimated to be 6.36 years, i.e. about 76 months. However, the probability of survival at month 76 calculated by the model is 0.023946507 , which means that only $2 \%$ of the whole population reaches the age of 76 months. Thus, the longevity of the stock seems overestimated by the classical model. As a future work, a probabilistic-based model for longevity could be devised using our approach.

Finally, an experiment that we performed was to determine how the fishing mortality could be reduced in order to have a survival probability greater than a certain value after a given number of months. This can be done by changing the probability constants and re-running the model on the period of interest. For instance, we found that a reduction of fishing mortality probabilities of $75 \%$ after 1 year and after 2 years (fishing probabilities in year 0 are already low because of the size of the fish) together with a reduction of $50 \%$ after 3 years, makes the survival probability after 4 years remain above 0.15 . These kinds of experiments are relatively easy to do on the model and have the potential of revealing new information regarding the impact of fishing regulations.

As future work we plan to derive more information both from the classical and the statistical model checking of our model. Moreover, we are working on the problem of how to correctly compare the different results that we get from the three approaches followed in Figure 1.

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[^1]:    ${ }^{5}$ The full PRISM model together with some variants with which we experimented is available at https://dl.dropboxusercontent.com/u/33462615/Soles.zip.

